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An Introduction to the Variable Inertia Flywheel (VIF)

A flywheel with variable moment of inertia, combining the functions of energy storage and power control is introduced. Potential designs are presented and the basic physical governing equations developed. Examples of the flywheel system powering a constant angular rate, frictional load, and an accelerating automobile are presented. Limitations of and potential for future development which become evident in the examples are discussed.

Introduction

Due to its very simplicity, the concept of storing energy in the rotating mass of a flywheel seems very attractive. A closer look, however, reveals subtleties which significantly complicate the implementation of an operational system. Primary among these complicating factors is the difficulty of transferring the energy to and from the energy storage flywheel.

The governing energy relation for a flywheel is

$$E = \left(\frac{1}{2}\right) I\omega^2 \tag{1}$$

where E is the energy content of the flywheel, I is the moment of inertia of the flywheel about the axis of rotation, and ω is its angular rate. Thus, in order to retrieve stored energy from a flywheel, the rotational rate must decrease. This decrease in rotational rate is not desirable as systems being powered by flywheels usually require a rotational rate which is constant or even increasing. This mismatch in rotational rate is usually compensated for by the use of a variable ratio transmission such as a traction drive, a hydraulic pump and motor, or an electric generator and motor. Each of these systems has drawbacks either in terms of cost, reliability, or most importantly, efficiency. Specifically, the electric and hydraulic transmissions (the most technically developed) may be inefficient and costly enough to hamper the practical development of commercial flywheel energy storage systems.

If the equation for stored energy is reconsidered, another approach to the problem becomes evident. It may be possible to alter the moment of inertia of the flywheel to gain added control of the release of stored energy. The resulting mechanism is called a Variable Inertia Flywheel (VIF). This mechanism is an energy storage device whose output can be controlled to meet the load requirements.

The design of a fixed inertia flywheel with a high energy density in terms of energy stored per unit weight and per unit volume is still a subject of much research. Present research is focused primarily in the area of the use of composite structures for flywheel rotors. The use of composites is giving new freedom to the design of flywheel shapes which are both more efficient and have the potential property of noncatastropic failure.

The design of VIF's will, by necessity, be more complex than that of a fixed inertia flywheel. To enable a flywheel to have a controllable moment of inertia about its spin axis, the geometry about that axis must be variable and to alter the geometry will require movable components in the flywheel. The increased complexity may result in a VIF weighing more per unit of energy stored than a fixed inertia flywheel, but if it is considered that the VIF is replacing both the flywheel and the transmission then the increased weight in the VIF may still result in a more energy dense system than its fixed inertia counterpart.

To investigate, on a preliminary level, the potential of VIF research this paper will present a brief discussion of potential VIF designs, a simple mathematical model describing the general operation of VIFs, and two examples of VIF applications.

VIF Designs

Potential VIF designs must be rotational mechanisms which are capable of varying their moment of inertia about the spin axis. They must, to be competitive with other energy storage systems, have high energy storage capability per unit weight, high moment of inertia change capability and be easy to actuate. Potential designs fall into two classes, fluid and mechanical. Although no direct individual analysis of any specific design has been undertaken here, some proposed configurations are presented in the paragraphs which follow. These are described in greater detail in reference [1].

Three fluid based VIF concepts appear in the patent literature. The most basic of these is the pumping of a fluid in and out of a hollow baffled casing [2]. Although there is much description of the casing and baffles in this reference, no mention is made on how to pump the fluid in and out of the flywheel. Another fluid concept [3] utilizes heavy magnetic solid particles in a nonmagnetic liquid matrix. The

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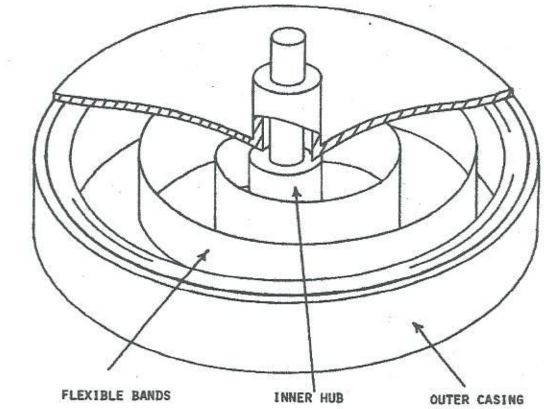

Fig. 1 Band-type variable inertia flywheel

central shaft is a controllable electromagnet and by varying the current in this shaft, the relatively heavy particles can be attracted toward the center, varying the moment of inertia. This invention was intended for low angular rate operation where the varying moment of inertia is required to alter the dynamic properties of an operating system, but it may be applicable to a VIF system. Reference [4] describes a spoked-rim flywheel where each spoke is a cylinder filled with a heavy fluid and a light piston. As the pistons are moved radially the moment of inertia is varied.

Mechanical flywheels which have been considered are the flyball governor, tilting rods, tilting disks, and a modification of the Brush-type Super-Flywheel [5] to allow drawing the brush filaments in and out of the central shaft. However the configuration which seems to have the qualities necessary for a successful system is the coiled band flywheel which is shown in Fig. 1. This flywheel is composed of a hollow outer casing and a separate central hub. Connecting these is a band of some flexible material mounted as the main spring in a watch. Centrifugal force pushes the band to the outer edge of the hollow casing. As the inner hub is rotated relative to the outer casing, the band is wound on the hub lowering the moment of inertia of the system. What is occurring is a balance between the centrifugal force on the band, the angular rate difference between the components and the torque flow through the mechanism supplemented by the torque variation caused by the change in angular momentum in the band. Work on this concept has appeared since an initial patent in 1965 [6]. Much of this work has been in the USSR with two patents [7, 8] and one published article in English [9]. However the authors have been unable to find any dynamic analysis of such a mechanism in the open literature. The dynamic balance of torques, forces, and momentum is surely complex. Understanding of the dynamics is required to fully understand the potential of the device. Present research is being conducted by the junior author into the exact potential and limitations of this configuration.

It is interesting to note that this concept is in complete concert with the anisotropic rim rotor Super-Flywheel [10]. The band is loaded uniaxially and in the high energy configuration (the band centrifugally pressed in the casing) the band resembles the rim motor.

Modeling the VIF

Independent of which type of VIF may prove most feasible, modeling the power flow can be performed without choosing a specific design, and the operational characteristics of the VIF in general can be investigated.

In order to describe mathematically the general VIF, three power flows must be considered: the power input from the external source, P_E (negative power is to a load); the power required to describe the total kinetic energy increase in the system, P_T; and the power required to vary the moment of inertia of the VIF; P_I (positive for decrease in moment of inertia). An assumption made in the following derivation is that the rate of effective mass radial position change is much smaller than the tangential velocity of the mass, $\dot{r} \ll r\omega$. The effect of this assumption has been shown to be negligible for systems with operational potential [1]. For example, doubling the energy level in a VIF in 1 sec, results in only 2 percent state variation between a full simulation and a simulation with the foregoing assumption. This assumption allows the total energy in the system to be approximated by

$$E \doteq \left(\frac{1}{2}\right) I\omega^2. \tag{2}$$

For any VIF the external torque on the VIF must be balanced by the change of angular momentum of the VIF. Thus

$$T_E = \frac{d}{dt}(I\omega) = \dot{I}\omega + I\dot{\omega} \tag{3}$$

and the power into the VIF supplied by this torque must be

$$P_E = \omega \frac{d}{dt}(I\omega) = \omega^2 \dot{I} + \omega\dot{\omega}I. \tag{4}$$

The total power required to increase the total kinetic energy in the system can be found by taking the time differential of the kinetic energy (equation (2)) resulting in

$$P_T = \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = \frac{1}{2}\dot{I}\omega^2 + \omega\dot{\omega}I. \tag{5}$$

Based on the definition of the positive directions of the powers made in the second paragraph, the total power increase (equation (5)) must equal the power from the external source (equation (4)) plus that power required to vary inertia. Thus

$$P_T = P_E + P_I. \tag{6}$$

With equations (4) and (5) then

$$P_I = \left(\frac{1}{2}\right) \dot{I}\omega^2 \tag{7}$$

Positive P_I means that power has been input into the VIF to reduce its moment of inertia. If the VIF is pictured as a mass on a radial string, it takes a power input to pull the string, reducing the radial position of the mass.

The aforementioned powers can also be represented as torques

Nomenclature

a = acceleration of automobile
E_a = energy used to accelerate automobile
E = energy content of flywheel
F = radial force required to change moment of inertia
I = moment of inertia of flywheel
I_f = fixed moment of inertia
m_e = effective mass of variable inertia
P_{AF} = power required to overcome automobile frictions
P_E = power flow in VIF due to change in angular momentum
P_I = power flow in VIF required for moment of inertia change
P_T = total power flow between VIF and load
r = radial position of effective mass
t = time
T_E = torque equivalent of time rate of change of angular momentum
T_I = torque equivalent of F
T_a = torque to accelerate automobile
T_T = total torque from VIF to load
v = velocity of automobile
ω = angular rate of flywheel
ω_a = angular rate of automobile wheels
$\dot{}$ = (superscript) derivative with respect to time
0 = (subscript) initial value

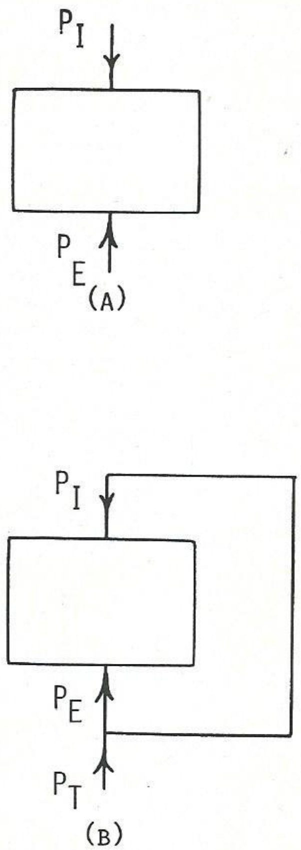

Fig. 2 Power flows in variable inertia flywheels: (*a*) open loop VIF power flow, (*b*) recirculating VIF power flow

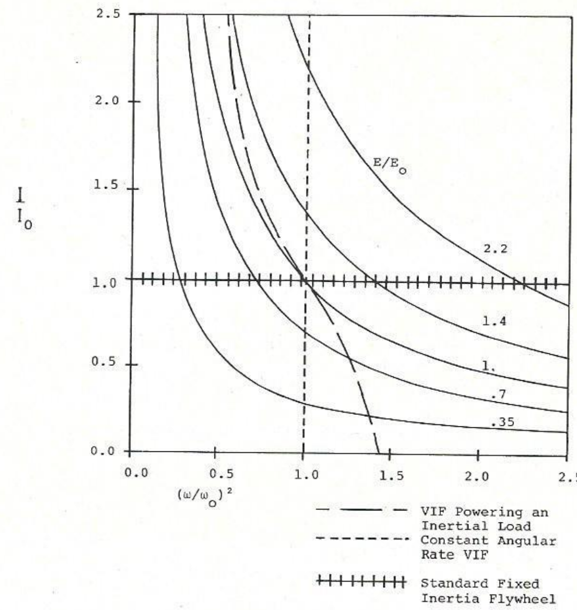

Fig. 3 Variable inertia flywheel isoenergy plot

where a positive torque is in the direction of rotation. The torque of the external load was given in equation (3). Also

$$T_T = \left(\frac{1}{2}\right)\dot{I}\omega + \dot{\omega}I \tag{8}$$

and

$$T_I = -\left(\frac{1}{2}\right)\dot{I}\omega. \tag{9}$$

Note that

$$T_T = T_E + T_I. \tag{10}$$

There are two approaches which can be taken in discussing the potential configuration power flow in a VIF. Consider the mechanism where the power for moment of inertia change comes from an outside source, Fig. 2(*a*), or consider the system as a whole with all the power required for state change entering through one port, the shaft to outside systems, as shown in Fig. 2(*b*). The consideration of the power for inertia change, P_I, coming from an outside source (open loop power flow) opens unlimited possibilities of another storage system providing the power, the power being supplied by a prime mover, or interaction between two VIF's. More comparable to a regular, fixed inertia flywheel however, is the case where the power flow in the output shaft is split between the load and the inertia change, that is part of the power recirculating through the VIF. Here power P_T is input to the VIF with some internal mechanism splitting this power between moment of inertia change and angular momentum change.

A basic insight into the character of the VIF can be had by examining the energy, equation (2). This equation is a function of two variables, I and ω, thus any energy level can be reached from an initial energy level through a change in rotational rate or a change in moment of inertia or a combination of changes in both. This can be shown by normalizing the energy equation with the initial state, I_0 and ω_0 so that

$$\frac{E}{E_0} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2. \tag{11}$$

Plotted in Fig. 3 are a few representative values of E/E_0 for a limited range of I/I_0 and $(\omega/\omega_0)^2$. Each curve in this figure will be called an isoenergy line as each is the locus of constant energy states. Also on this plot are shown the locus of possible energy states for a standard fixed inertia flywheel (only ω is variable), a VIF maintaining a constant angular rate, (discussed in the next section) and a VIF powering an inertial load where the load has the same moment of inertia as the VIF does initially [1].

The VIF With a Constant Angular Rate Load

The dynamic equations just derived readily reduce to a rather basic law of VIF operation when considered for a constant angular rate. In this case equations (4) and (7) reduce to

$$P_E = \omega^2\dot{I} = -2P_I, \tag{12}$$

and equation (5) reduces to

$$P_T = \left(\frac{1}{2}\right)\omega^2\dot{I} = -P_I. \tag{13}$$

Thus, for a closed loop configuration, one-half the power released by the flywheel due to momentum change must be recirculated to change the inertia. This is an extremely important conclusion and is totally independent of the physical design of the VIF. Further, from the torque relations, the rate of inertia variation is uniquely prescribed as a function of the varying load torque and constant angular rate,

$$\dot{I} = 2T/\omega^2. \tag{14}$$

If $\dot{I}$ and be controlled to satisfy equation (14) then the load demanding torque T, will be powered at constant angular rate, ω.

The VIF as Applied to an Accelerating Automobile

Probably the most provocative loading for a flywheel energy storage system is the case of an accelerating automobile. The VIF as applied to this loading is assumed, as it has been throughout this paper, to be ideal with no internal losses. The same will be true of the remainder of the drive train. However, the automobile will be more than an inertia load as the model will include aerodynamic and tire drag.

The automobile modeled [11] weighs 6670 N(1500 lb) and is accelerating with velocity

$$v = 30.5(1 - e^{-0.051t})\ \text{m/sec}, \tag{15}$$

and the acceleration is

$$a = 1.55e^{-.051t}\ \text{m/sec}. \tag{16}$$

These imply that the initial acceleration rate is about 1/6 times the acceleration of gravity and the acceleration diminishes exponentially to final top speed of 109 km/hr (68 mph).

If the wheel radius is assumed as 0.305m (1 ft), then the velocity of the automobile is equivalent to the angular rate of the wheel, ω_a. Thus the angular rate time history of the wheel is

$$\omega_a = 30.5\ (1 - e^{-0.051t})\text{rad/sec}. \tag{17}$$

From the acceleration given in equation (16), the force and thus the power required for accelerating the mass can readily be calculated. This result characterizes that load which would be an inertial load (acceleration forces only) on the VIF but, for a more complete picture, aerodynamic and tire drag losses (velocity-dependent forces) must be included.

From reference [11] the power lost to air and tire friction is

$$P_{AF} = 2.68v[(0.0001395v + 0.01676)m_e + 0.022128v^2]\text{watt}. \tag{18}$$

Combining the inertial and drag power equations gives the resulting power required to accelerate the vehicle as

$$P_T = 20385\ (1 - e^{-0.051t}) + 14688\ (e^{-0.102t} - e^{-0.153t})\ \text{watt} \tag{19}$$

Integrating this over time yields the energy used by the vehicle,

$$E_A = 1356\ (15.1t + 296 \times e^{-0.051t} - 106.7e^{-0.102t} + 71.1e^{-0.153t} - 260.4)\ \text{Joule} \tag{20}$$

To complete the set of equations necessary to fully describe the vehicle, the torque required for the acceleration is P/ω, so using equations (17) and (19) results in

$$T_a = 204.7 + 147.5^{-0.102t}\text{N} \cdot \text{m}. \tag{21}$$

To power the vehicle through this acceleration, a VIF must be sized which stores enough energy to accomplish the task. As this discussion is merely an example and not meant to be optimum, the following VIF parameters will be used as convenient;

$$I_{\max} = 5.42\ \text{N} \cdot \text{m} \cdot \text{sec}^2 (4.\ \text{lb} \cdot \text{sec}^2 \cdot \text{ft})$$

$$I_f = 0.0\ \text{N} \cdot \text{m} \cdot \text{sec}^4$$

$$\omega_{\max} = 160.5\ \text{Hz}\ (100\text{rad/sec}).$$

The initial energy content of the VIF is 664 kilojoules which will power the vehicle for about 43 sec. This energy is expended for a maximum acceleration for these 43 sec; a very demanding task.

Finally, there will be assumed a fixed ratio gear box between the VIF and the wheels. This unit includes any differential required. It will be assumed that the gear ratio is R where

$$\omega = \omega_a R \tag{22}$$

and

$$T_T = T_a/R. \tag{23}$$

This ratio will be variable in discrete steps as in a standard automobile transmission.

Before continuing, it must be emphasized that the following analysis is only to show the nature of what must happen in a VIF powered automobile; thus some of the simplifying assumptions would not be practical in an operational system.

Based on the aforementioned equations and assumptions, the flywheel rotational rate and moment of inertia are uniquely determined as functions of time and gear ratio. Thus, from equations (17) and (22),

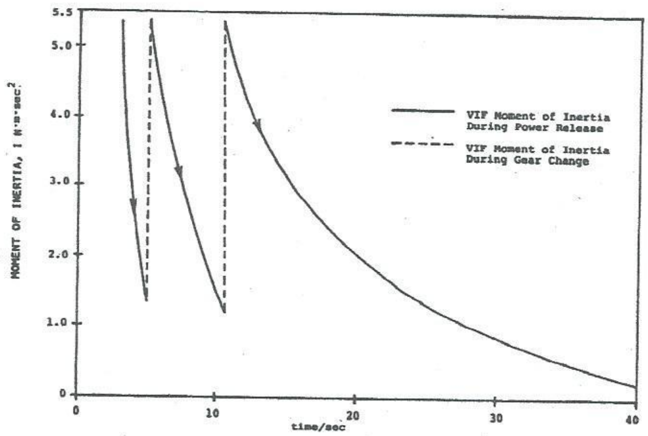

Fig. 4 Moment of inertia of the variable inertia flywheel powering an accelerating automobile

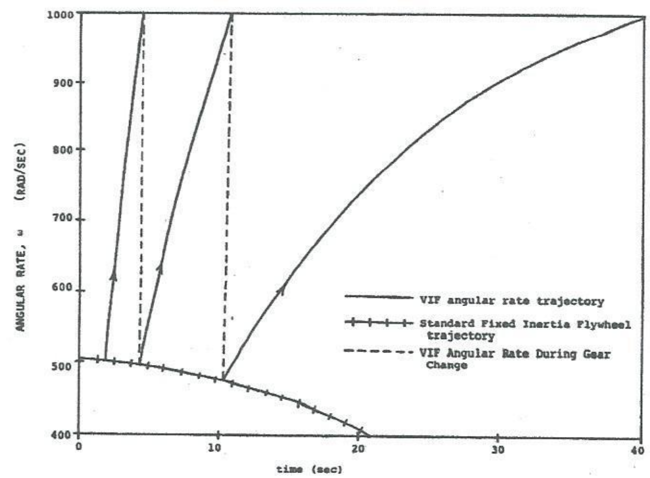

Fig. 5 Angular rate of the variable inertia flywheel powering an accelerating automobile

$$\omega = 30.5(1 - e^{-0.051t})R\ \text{rad/sec} \tag{24}$$

The moment of inertia time history can be found by taking the initial energy level of the VIF, E_0 and subtracting the energy required to accelerate the automobile, equation (20). This results in

$$I = \frac{2}{\omega^2} \cdot \frac{2(E_0 - E_a)}{\omega^2}\text{N} \cdot \text{m} \cdot \text{sec}^2. \tag{25}$$

These equations give the time history of I and ω where R is assigned. Let R be initially assigned by taking $\omega_0/\omega_{a0'}$ the ratio of initial rates. This value will be used until the VIF angular rate has increased to $\omega_{\max}$ at which time a new gear ratio will be selected. This new ratio will be calculated in such a way that the VIF has the same energy level before and after shifting but, after shifting the effective moment of inertia is maximum and the angular rate is reduced somewhat from its maximum value. At time zero there is a problem of requiring R to be infinite. This is overcome by assuming a slipping clutch or torque converter for the first 2 sec so the initial conditions on the automobile's wheels angular rate is $\omega_0 = 9.7$ rad/sec.

The results of a computer simulation for the system, the VIF moment of inertia, the VIF angular rate and the gear ratio required are plotted in Figs. 4–6. Also shown on these plots is the time history of a fixed inertia flywheel of the same mass acting through some type of variable ratio transmission. Note that this solution acts as a bound for the lower mass VIF solution. From Fig. 4 it is seen that the VIF starts at 500 rad/sec and increases rate until 1000 rad/sec is reached

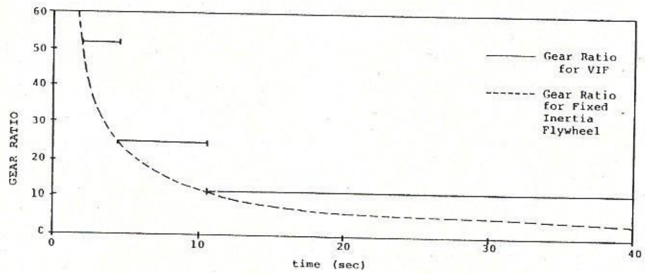

Fig. 6 Gear ratio of transmission for the variable inertia flywheel powering an accelerating automobile

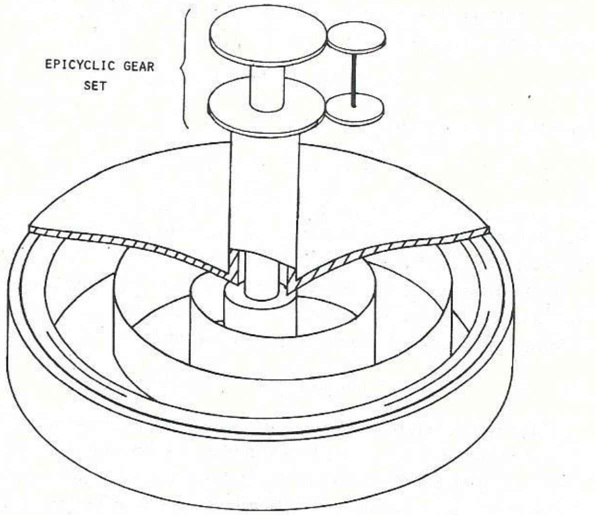

Fig. 7 Fixed ratio power recirculation variable inertia flywheel

then the gear ratio is changed. As can be seen in the VIF torque relations, the rate of decrease in mass radius or moment of inertia must create a torque equal to the torque required by the automobile plus that due to the angular rate increase. This torque generation during periods of high torque demand and high acceleration rates is extremely demanding as both high torque and angular acceleration require a high rate of decrease in mass radius or moment of inertia.

The primary conclusion which can be drawn from this example is that the VIF performs very well with a predominantly friction load (time >5 sec). The high acceleration required when the loading is primarily inertial (time <5 sec) makes extreme demands on the VIF.

Limitations of VIF Design

As can be seen in the previous examples the dependence of the inertia level and rate of change varies with the characteristics of load. Thus it would be most desirable to have the recirculating power flow to the inertia change, P_I, be completely controllable to allow for differing inertia change requirements. Unfortunately, complete controllability of the recirculating power, P_I, can only be had with a variable ratio transmission such as that which the VIF is proposed to eliminate. Also, it was shown that with constant rotational rate loading, the amount of power to be recirculated through the variable ratio transmission would be the same as that output. Thus it appears that the VIF in this configuration could have no better efficiency than a fixed inertia flywheel with the variable ratio transmission on the output shaft.

However, complete control of the recirculating power is not required for most tasks as the nature of the loading is usually reasonably well known and slight deviations from the desired are usually tolerated; thus it is possible to create a fixed power recirculation path to meet the requirements of a specific load. Use of the term "fixed power recirculation" implies that the mechanism carrying power P_I to create the inertia change is of fixed geometry or at a maximum countably few variations in geometry (e.g., a fixed ratio transmission). The result of this VIF with a fixed path, is a flywheel with fixed properties which differ from those of a standard fixed inertia flywheel. Another way of stating this is that now the moment of inertia is a function of angular rate so that the torque is now totally a function of angular rate but different from that of a standard fixed inertia flywheel. An example of such a VIF with fixed ratio power recirculation is shown in Fig. 7. This is a band-type VIF with the inner and outer casings connected through an epicyclic gear set. In this example the exact paths of P_F and P_I are not distinct but the power recirculation loop is defined by the eplicyclic gear ratio and the band/casing geometry. Limitations on this concept are being studied in conjunction with the analysis of the band-type VIF described previously. Specifically, the configuration in Fig. 7 is being pursued as a first step with strong emphasis being given to integration in practical systems.

Conclusions About the Basic Characteristics of VIF

From the preceding analysis the following general conclusions can be drawn:

1 Power is transferred to or from the VIF by two mechanisms; angular rate change and moment of inertia change. Power transfer due to angular rate change is as in a fixed inertia flywheel, $\omega\dot{\omega}I$, and power transfer due to moment of inertia change is $(1/2)\,\omega^2\dot{I}$, equation (7).

2 In order for the VIF to act as other than a fixed inertia flywheel, the change in moment of inertia must be continuous.

3 The VIF can release torque, producing energy, while increasing in rotational rate. The power released by lowering the moment of inertia is used to both accelerate the VIF and to provide power to the load.

4 The assumption that the forces on the system due to inertia change acceleration are far smaller than those forces due to centrifugal, angular and coriolis accelerations is supportable.

5 The power required for inertia change is equal to the output power for a constant rotational rate power recirculation VIF. For other loadings the power required for inertia change is the same order of magnitude as the output power.

6 A fully controllable VIF, full control over the moment of inertia irrespective of the VIF state, is not practical as the efficiency obtained can be no better than a fixed inertia flywheel with an infinitely variable ratio transmission.

7 A VIF with a fixed power recirculation results in a flywheel with a unique torque/angular rate characteristics. Whereas a fixed inertia flywheel is restricted to $T_f = I\dot{\omega}_f$, a VIF can theoretically be formed such that the torque is related to any function of angular rate.

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