K. H. Lim
Hankuk Aviation College,
Kyungki-Do, Korea 411-910

D. G. Ullman
Associate Professor.
Department of Mechanical Engineering,
Oregon State University,
Corvallis, OR 97331

Optimal Design of Traction Drive Continuously Variable Transmissions

An optimal design technique for minimum power loss in traction drive continuously variable transmissions is developed. The general forms of the objective function and constraint equations are derived, and the formulated optimal design problems are implemented in a nonlinear programming algorithm. Kinematic analysis and optimal design problem formulation are performed for a selected traction drive configuration as an example of the procedures.

1 Introduction

A traction drive is a transmission device that can transfer power by means of metal rollers running on a film of traction fluid. Traction drives can be designed not only as fixed speed ratio transmissions, but also as variable ratio transmissions. A traction drive that can change speed ratio continuously is called traction drive continuously variable transmission (TD-CVT). TD-CVTs have many advantages over gear drives. Of course, the main advantage is that the speed can be continuously varied to maximize the available power from a power source. Additionally they run quietly, have minimal vibration, and are easy to manufacture because of circular-cross section components.

Despite the advantages, the overall efficiency of most TD-CVTs is lower than that of gear drives. Thus it is important to design TD-CVTs with maximum efficiency. The goals of this paper are to develop an optimal design technique for traction TD-CVTs, and to investigate the optimality of an existing traction drive configuration.

To achieve the goals, the following procedures are employed. Relationships between the geometrical parameters are derived through a kinematic study on the chosen configuration. The equation for power loss is derived as the objective function to minimize. The constraint equations are derived from the balance of power and conditions of a traction drive contact. The formulated optimal design problem is solved using a nonlinear programming algorithm called Generalized Reduced Gradient (GRG) method.

2 Kinematics of Traction Drive Contact

The kinematics of traction drive contact can be understood as composed of two touching rolling elements. The traction force in the contact is dictated by the amount of spin and slip at the contact. The amount of slip depends on not only the geometry of the traction drive but also the torque load and the applied normal force at the contact. The spin, however, can be determined from the geometry of a traction drive alone. Spin generally occurs in traction drives using rollers with non-parallel rotation axes. This feature is inevitably present in all traction drive CVTs that change configuration for speed variation. Spin causes power loss, reduces available traction force, and generates traction forces perpendicular to the rolling direction. Thus, spin has an ill effect on traction drives and should be kept as small as possible to have best performance [5, 6].

The general contact geometry in a traction drive is shown in Fig. 1. The angular velocities of Roller 1 and 2 are given as ω_1 ω_2. Angular velocities perpendicular to the tangential contact plane are $\omega_1 \sin\alpha$ and $\omega_2 \sin(\beta-\alpha)$ on Roller 1 and Roller 2, respectively, where α is the angle between the rotation axis and the tangential contact plane and β is the angle between the two rotation axis. Spin, the relative angular velocity difference perpendicular to the contact area, is ω_s, and can be expressed as

$$\omega_s = \omega_1 \sin\alpha - \omega_2 \sin(\beta-\alpha). \tag{1}$$

For an idling condition, that is when there is no torque in the system thus no tangential force, there is no slip across the contact, and

$$\omega_1/\omega_2 = R_2/R_1 \tag{2}$$

Rewriting (1) as

$$\psi = \sin\alpha - A\sin(\beta-\alpha) \tag{3}$$

where $\psi = \omega_s/\omega_1$
and $A = R_1/R_2 = \omega_2/\omega_1$
R_1, R_2: rolling radii of rollers.
In equation (3) ψ is the ratio of spin to input angular velocity. This ratio is used to describe the kinematic relations between two rollers [14, 15]. Additionally, A is the ratio of two roller's angular velocities.

In general, R_1, R_2, α, and β vary with changing speed in a traction drive CVT configuration. Thus ψ changes also. For pure rolling, ψ is equal to zero. Pure rolling is the ideal condition for the minimum power loss in a traction drive.

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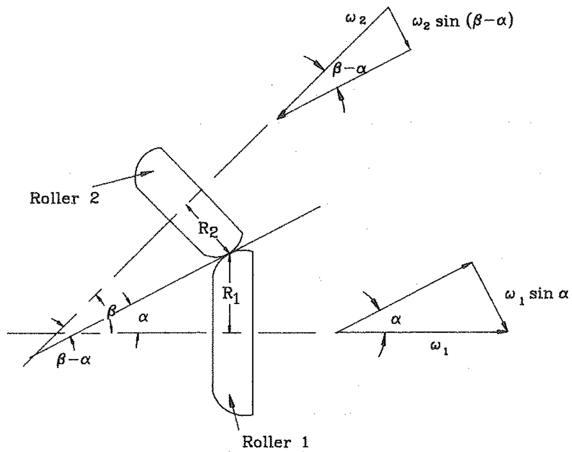

Fig. 1 Spin geometry in a traction drive contact

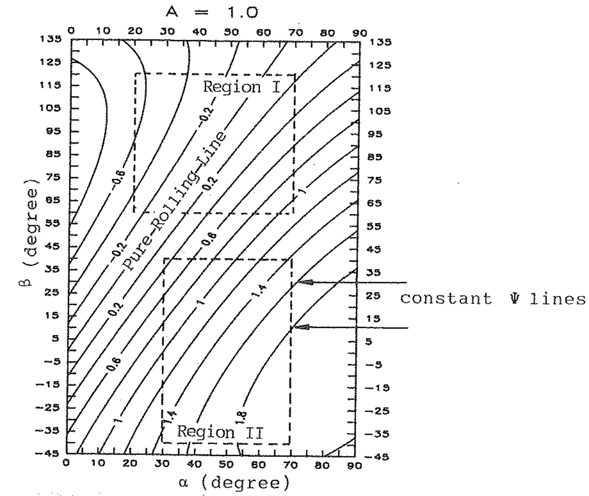

Fig. 2 Constant ψ lines for $A = 1.0$

Therefore, a near pure rolling traction drive configuration is generally optimal in terms of minimum power loss during power transmission. To find the minimum spin configuration, equation (3), the locus of $\psi(\alpha, \beta)$, is plotted as shown in Fig. 2 for $A = 1$. Similar plots for other A values can be found in [4].

In Fig. 2, Region I represents the contact geometry 20 deg $< \alpha < 70$ deg, and 60 deg $< \beta < 120$ deg. One traction drive configuration for this region can be drawn as in Fig. 3(*a*), the Toroidal Traction Drive [4]. In this region ψ is in the range from roughly -0.6 to 1.0.

In Region II, where 30 deg $< \alpha <$ 70 deg and -40 deg $< \beta <$ 40 deg, the range of ψ is roughly from 0.4 to 1.8, which is higher than in Region I. In this region, traction drive configuration can be drawn as in Fig. 3(*b*), a Ball Variator.

The kinematics of the Region I configuration (i.e., the Toroidal Traction Drive) will be used as an example in this paper because it has relatively small values of ψ. Figure 4 shows the geometry and spin diagrams of the Toroidal Traction Drive. In this configuration there is slipping that shears the traction fluid to produce a traction force and transmit power at both input and output contact points. The slip at the input contact ($SLIP_i$) and the slip at the output contact ($SLIP_o$) are given by

$$SLIP_i = (\Delta U/U)_i$$
$$SLIP_o = (\Delta U/U)_o \qquad (4)$$

Where

ΔU = tangential velocity difference at contact
U = average tangential velocity at contact
subscript $_i$ = input contact
subscript $_o$ = output contact

The velocity of the middle roller at the input contact (U_{mi}) and at the output contact (U_{mo}) can be written as

$$U_{mi} = U_i(2 - SLIP_i)/(2 + SLIP_i)$$

and (5)

$$U_{mo} = U_o(2 + SLIP_o)/(2 - SLIP_o).$$

The angular velocity of the middle roller is $\omega_m = U_{mi}/R_m \sin\theta$. Based on this, the spin at the input contact (ω_{si}) and the spin at the output contact (ω_{so}) can be found from the spin diagram in Fig. 4.

$$\omega_{si} = \omega_i \sin(\theta - \phi) - \omega_m \cos\theta$$
$$\omega_{so} = \omega_o \sin(\theta + \phi) - \omega_m \cos\theta \qquad (6)$$

The inclination angle ϕ can be evaluated if the speed ratio

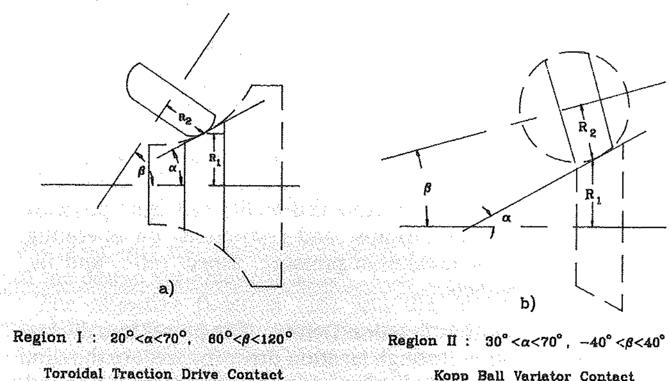

Fig. 3 Traction drive configuration for Region I and II

SR (ω_o/ω_i), $SLIP_i$, $SLIP_o$, and other basic dimensions R_m, D, θ, are known as [5]

$$\phi = \sin^{-1}\left\{\frac{-q - q^2 - (p^2 + q^2)(1 - p^2)}{p^2 + q^2}\right\} \quad \text{when } SR' < 0$$

$$\phi = \sin^{-1}\left\{\frac{-q + q^2 - (p^2 + q^2)(1 - p^2)}{p^2 + q^2}\right\} \quad \text{when } SR' > 0$$

(7)

Where

$$p = \frac{R_m \cos\theta}{D}$$

$$q = \frac{R_m(SR' + 1)}{D(SR' - 1)} \sin\theta$$

and $SR' = SR/S$

where

$$S = \frac{(2 - SLIP_o)(2 - SLIP_i)}{(2 + SLIP_o)(2 + SLIP_i)}$$

and S depends on only $SLIP_i$ and $SLIP_o$. For idle operation S is equal to one because there is no slip, and for normal operation S is less than one because slip always occurs at the contact areas.

In the next section, the optimal design problem is formulated for minimum power loss in a traction drive *CVT*. Again the Toroidal Traction Drive will be used as an example.

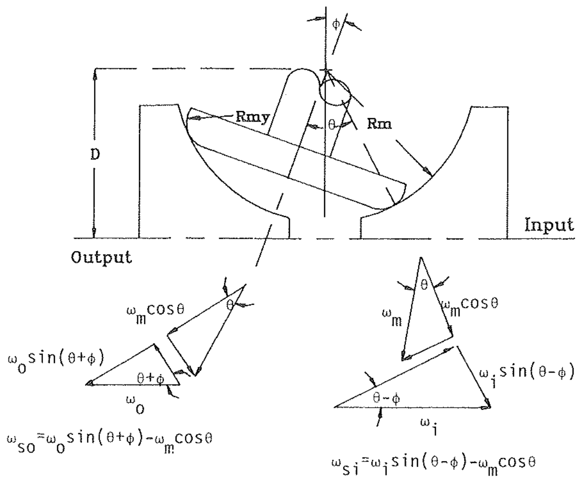

Fig. 4 Spin diagram and geometry of the toroidal traction drive

3 The Optimal Design of a Traction Drive CVT

There are two major sources of power loss during power transfer in a traction drive CVT. One is the power loss due to spin and slip at the traction drive contact, and the other is power loss due to bearing friction torque. It is assumed that the windage loss is negligible [15]. The sum of the two power losses is the objective function to minimize. The constraint equations for the traction drive can be derived from physical laws, geometric relationships, and restrictions on operating conditions such as maximum pressure, aspect ratio, and fatique life of contacts.

3.1 Power Loss in a Traction Drive. The power loss during power transmission through traction drive contact are the sum of power losses due to slip in the rolling direction $F_x \Delta U$ and power loss due to spin $T\omega_s$. F_x is the traction force and ΔU is the relative velocity in rolling direction. T and ω_s are the torque and spin perpendicular to the contact area, respectively. F_x and T will be calculated numerically from Johnson and Tevaarwerk's traction model [4, 11, 12]. There is no power loss due to side slip because it is assumed that there is no side slip which would be caused by misalignment. Therefore, the power loss at contact, L_c, can be written as

$$L_c = F_x \Delta U + T\omega_s. \tag{8}$$

From Johnson and Tevaarwerk's traction model,

$$F_x = \mu N J_4 \tag{9}$$

$$T = \mu N \sqrt{ab} J_6 \tag{10}$$

where J_4 and J_6 are the nondimensional force and torque at the contact and defined as $J_4 = \mu_x/\mu$, $J_6 = T/(\mu N\sqrt{ab})$ in [4, 11, 12]. Here μ is the maximum available traction coefficient, μ_x is the traction coefficient, and N is applied normal force at contact. Also a and b are the semi-widths in the transverse and rolling directions of the contact ellipse. Substituting the above equations in (8)

$$L_c = J_4 \mu N \Delta U + J_6 \mu N \sqrt{ab}\,\omega_s$$

$$= J_4 \frac{\mu N U}{C} \frac{C \Delta U}{U} + J_6 \frac{\mu N U}{C} \frac{C\omega_s \sqrt{ab}}{U}$$

or

$$L_c = \mu N U (J_4 J_1 + J_6 J_3)/C \tag{11}$$

where C is defined as

$$C = \frac{3\pi}{8} \frac{m}{\mu} \sqrt{k}$$

and m is the initial slope of the traction curve. J_1 and J_3 are the nondimensional slip and spin, respectively, as defined in [4, 11, 12]: $J_1 = C\Delta U/U$, $J_3 = C\omega_s \sqrt{ab}/U$. Equation (11) is the power loss due to spin and slip at a traction drive contact.

The power loss due to bearings in a traction drive cannot be ignored because of the relatively high power loss which originates from the high normal load at the traction drive contact. The high normal force is needed to transmit power without gross slip. The bearing loss can be evaluated from the bearing torque friction equation. The friction torque in bearings can be evaluated [1, 3] from equation

$$T_B = T_f + T_1 \tag{12}$$

where T_f is the load-free friction torque and T_1 is the load dependent component of the total friction torque. The values of T_f and T_1 can be evaluated from [1, 3] the equations

$$T_f = 1.42 \times 10^{-5} f_0 (v' n)^{2/3} d_m^{\,3}$$

and

$$T_1 = f_1 F_\beta d_m$$

where f_0 is the coefficient depending on the type of bearing and the method of lubrication, v' is the kinematic viscosity of the lubricant in centistokes, n is rpm of shaft, d_m is the pitch circle diameter of bearing, f_1 is a factor depending on bearing design and relative bearing load, and F_β depends on magnitude and direction of the applied load. The power loss for each bearing can be calculated from the equation

$$L_B = T_B \omega \tag{13}$$

where ω is the angular velocity of the shaft where the bearings are located.

3.2 Problem Formulation for Optimization. The objective function is the total power loss in a traction drive during power transmission. The total power loss is the sum of the losses at the contacts and losses due to bearings. The total power loss due to the contact for each middle roller is

$$L_C = (L_i + L_o)$$
$$= \mu_i N_i U_i \quad (J_{4i} J_{1i} + J_{6i} J_{3i})/C_i$$
$$+ \mu_o N_o U_o \quad (J_{4o} J_{1o} + J_{6o} J_{3o})/C_o,$$

Where L_i is loss at input contact, and L_o is loss at output contact. Thus the total power loss in any traction drive is

$$L = M L_C + \sum_{t=1}^{N_B} L_{Bt}, \tag{14}$$

Where M is the number of the middle rollers, L_{Bt} is the bearing loss, and N_B is total number of bearings in the traction drive. Equation (14) is the objective function to minimize.

The equality constraints can be derived from the balance of power. The power at the input contact P_i' is the power input from power source minus the bearing loss at the input side as shown in Fig. 5.

$$P_i' = P_i - L_{Bi} \tag{15}$$

where P_i is the power from the power source such as the motor or engine, and P_i' is the actual power at the input contact to the traction drive. This constraint effectively defines the amount of power input to the traction drive. The other constraint comes from the condition that there is no power loss in the middle roller except for the bearing loss.

$$P_{om} = P_{im} - L_{Bm} \tag{16}$$

Equations (15) and (16) are the equality equations.

The inequality constraint equations come from restrictions on the following parameters; aspect ratio, $k = a/b$; maximum

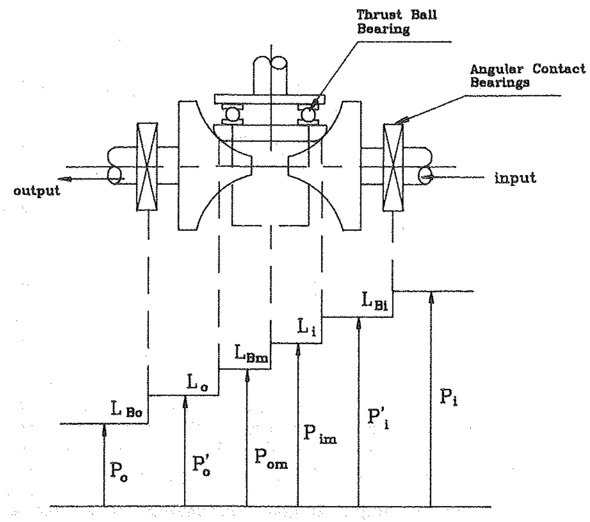

Fig. 5 Power loss at contact and bearings for the toroidal traction drive

contact pressure, p; nondimensional spin, $\text{spin} = \omega_s \sqrt{ab}/U$; and average tangential velocity, U. The upper and lower bound of these parameters were given as

$$0.3 < k < 8.0 \tag{17}$$

$$1.0 < p < 2.5 \text{ GPA} \tag{18}$$

$$0.0 < SPIN < 0.04 \tag{19}$$

$$0.0 < U < 100.0 \text{ m/sec} \tag{20}$$

because these bounds are the typical limits on a traction drive contact. The condition

$$0.0 < J_4 < 0.9 \tag{21}$$

is added to the inequality constraints, where the upper bound allows some room for gross slip when an abrupt load is applied. Also a constraint is given for the fatigue life of the traction drive contact surfaces because fatigue failure may occur due to fluctuating subsurface stresses. The fatigue life equation is given by Loewenthal and Zaretsky [8] as

$$FL = 2.32{*}10^{19} K_2^{\ 0.9} N^{-3} \rho^{-6.3} R_r^{\ -0.9}$$

where FL is 90 percent survival life of a single contacting element in millions of stress cycles, K_2 is a parameter related to curvature of rollers, and ρ and R_r are parameters related to the radius of the rollers. The lower limit on FL is set at 1000 and there is no limit on the upper bound of FL. Therefore the constraint equation for fatigue life is

$$1000 < FL \tag{22}$$

A 1494 Watt (2 hp), 1750-rpm input to a Toroidal Traction Drive is selected as an example of optimal design. Two angular contact bearings were selected for both of the input and output side of the unit and there is one thrust bearing on the middle roller. The number of middle rollers is assumed as 2. Therefore the objective function (14) becomes

$$L = 2L_C + \sum_{t=1}^{6} L_{Bt} \tag{23}$$

Equations (15) and (16) can be expressed as

$$1494 - L_{Bi} = 2F_{xi} U_i = 2J_{4i} \mu_i N_i U_i \tag{24}$$

and

$$J_{4i} \mu_i U_{mi} N_i - L_{Bm} = J_{4o} \mu_o U_{mo} N_o. \tag{25}$$

J_4 and J_6 are functions of J_1, J_3, and k. J_1 and J_3 can be evaluated if N, $SLIP_i$, $SLIP_o$, and dimensions of the toroidal traction drive are known [4, 5] as

$$J_4 = J_4(J_1, J_3, K)$$

$$= J_4(N, SLIP_i, SLIP_o, \theta, D, R_m, R_{my})$$

and

$$J_6 = J_6(J_1, J_3, k)$$

$$= J_6(N, SLIP_i, SLIP_o, \theta, D, R_m, R_{my}).$$

Therefore, the objective functions (23) and constraints equations (24) and (25) are functions of the dimensions of the traction drive θ, D, R_m, R_{my}, and operating condition N, SLIP_i, and $SLIP_o$. The geometric parameters (θ, D, R_m, R_{my}) are shown in Fig. 4, where R_{my} is the radius of the middle contact as in Fig. 4. These seven variables are design variables for the Toroidal Traction Drive. The objective function L can be evaluated if the seven design variables are known for given input power, input rpm, nominal speed ratio, and bearings as

$$L = L(N, SLIP_i, SLIP_o, \theta, D, R_m, R_{my}) \tag{26}$$

Also the equality constraints equations can be evaluated if the values of the design variables are known as

$$1494 - L_{Bi} - 2J_{4i} \mu_i N_i U_i$$

$$= g_1(N, SLIP_i, SLIP_o, \theta, D, R_m, R_{my}) = 0 \tag{27}$$

and

$$J_{4i} \mu_i U_{mi} N_i - L_{Bm} - J_{4o} \mu_o U_{mo} N_o$$

$$= g_2(N, SLIP_i, SLIP_o, \theta, D, R_m, R_{my}) = 0. \tag{28}$$

The inequality constraints (17) through (22) are also functions of the design variables (N, $SLIP_i$, $SLIP_o$, θ, D, R_m, R_{my}.

The objective function (26) and the equality constraint equations (27) and (28) cannot be expressed explicitly by the design variables. Rather they must be evaluated using a numerical method because the analytic solution of the governing equation of the Johnson and Tevaarwerk's traction model is not known [4, 12].

4 Results of the Optimal Design Example

The Generalized Reduced Gradient (GRG) [2, 9, 10, 13] method is used to minimize the power loss for a 1494 watt (2 hp) power, and 1750 rpm input to the Toroidal Traction Drive. The optimization problem was implemented in a GRG code called OPT [2]. The design speed ratio (SR) 1.0 was chosen based on the assumption that SR = 1.0 is the most frequently used speed ratio. The calculated minimum power loss is $L = 68.65$ Watts. The optimum operating condition is $N = 2390.64$ Newton, $SLIP_i = 0.0084$ and $SLIP_o = 0.0074$. The physical dimensions are $\theta = 45.040$, $D = 4$ cm, $R_m = 3$ cm, and $R_{my} = 2.5$ cm.

Using these dimensions the optimal conditions at speed ratios other than unity can be determined. Here the number of design variables reduces to only three (N, $SLIP_i$, $SLIP_o$). With the reduced optimization problem, the optimum power loss for a 1494 watt power input toroidal traction drive is found for speed ratio range from 0.5 to 2.4. The efficiency of the designed traction drive can be calculated from η = (input power-loss)/input power = (1494-loss)/1494 and is plotted in Fig. 6. Figure 6 shows that the efficiency of the optimally designed unit is over 95 percent throughout the range.

The ratio of spin to angular velocity at input contact, ψ_i, is plotted versus speed ratio as shown in Fig. 7. In the figure, ψ_i is very close to zero. Thus, near pure rolling is achieved at the input contact throughout the range of the Toroidal Traction Drive speed ratios.

In commerical traction drives there exists a device that controls the applied normal force depending on output torque to prevent gross slip at contact. The relationship of the output

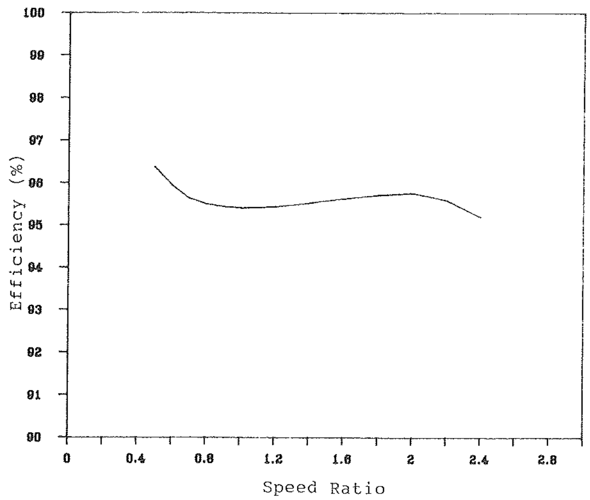

Fig. 6 Efficiency of a 1494 watt toroidal traction drive

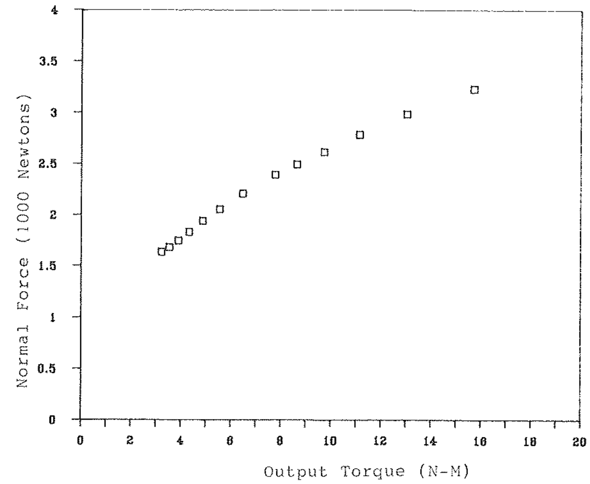

Fig. 8 Normal force—output torque relationship for a 1494 watt toroidal traction drive

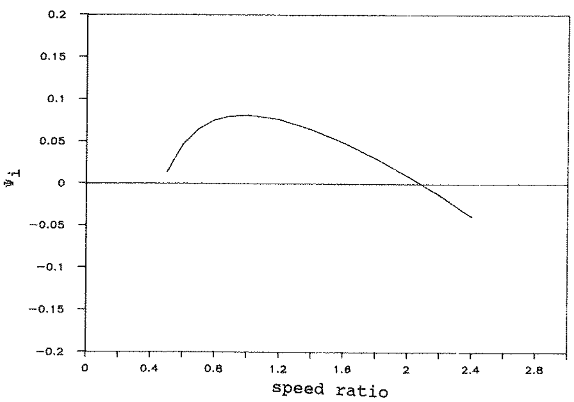

Fig. 7 ψ_i for a 1494 watt toroidal traction drive

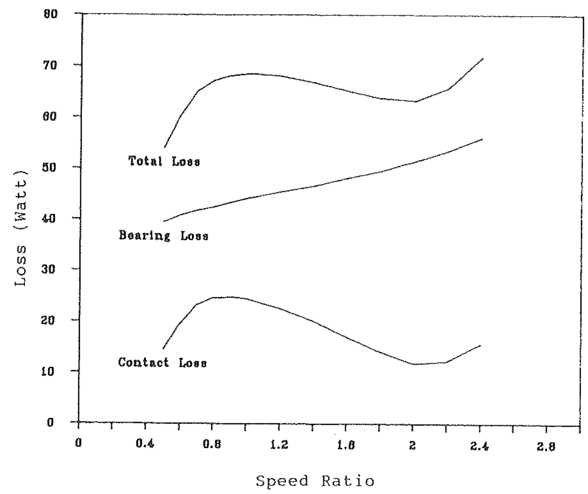

Fig. 9 Bearing and contact losses for a 1494 watt toroidal traction drive

torque to normal force as found in the optimization is given in Fig. 8. This shows that the relationship is almost linear. The linear relationship between normal force and output torque is easily accomplished by wedge-action mechanism.

The power loss occurs in two ways: one is due to slip and spin at contact and the other is due to bearing loss. Figure 9 shows that power loss due to bearing loss is relatively higher. Power loss due to slip and spin is relatively small because the spin is so small at the contacts in the toroidal traction drive.

Conclusions

An optimal design technique for traction drives has been developed. It has been demonstrated on the Toroidal Traction Drive.

5.1 Optimal Design Technique. In general, with the nominal input power, input angular velocity, and operational range of speed ratio given, the optimal design technique for any traction drive CVT can be summarized as follows.

Step 1: Choose a traction drive configuration.

Step 2: The spin equations at traction drive contacts should be derived from a kinematic analysis of the configuration. Equations (6) are the spin equations at input and output contact for the Toroidal Traction Drive. Similar equations can be developed for other configurations. The speed changing parameter—for example, the inclination angle d for the Toroidal Traction Drive— must be expressed in terms of speed ratio SR, dimensions of the traction drive, and the slip at the contacts as in equation (7) for the Toroidal Traction Drive.

Step 3: Formulate the optimal design problem in terms of design variables as in equations (26), (27), and (28), and then implement the problem in a nonlinear programming algorithm. The design variables consist of the dimensions of the traction drive, the applied normal force, and the slip at the contacts.

The objective function is the total power loss which is the sum of power loss at contacts and power loss due to bearing friction torque in a traction drive. The traction force and torque at a traction drive contact can be evaluated from the numerical solution of the mathematical model [4, 11, 12]. Then the power loss at a contact, equation (11), gives the power loss due to spin and slip at a traction drive contact. The power loss in a bearing can be found from equations (12) and (13). The general form of objective function is given as equation (14).

Equality constraints are from the balance of power prior to the input contact and at middle roller or middle ball. For the Toroidal Traction Drive, the equality constraints are given as equations (15) and (16). Inequality constraints are from the restrictions on the operating conditions of a traction drive contact such as maximum pressure, aspect ratio of the contact shape, etc. In general, equations (17) through (22) can be used as inequality constraints. The formulated optimal design problem needs to be implemented in a nonlinear programming algorithm.

Step 4: Find the optimum dimensions at the selected design speed ratio for minimum power loss using a nonlinear programming algorithm.

Step 5: Since the optimum dimensions are found in Step 4, the design variables are reduced to applied normal force and slip at contacts. Optimize the applied normal force over the other speed ratios for minimum power loss.

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